Comparison of Two means

Perhaps one of the most common problems in statistical inference is a comparison of two population means using data from two independent samples ; the sample sizes may or may not be equal. In this type of problem, we have two sets of continuous measures, one of size n_1 and one of size n_2 , and we consider the null hypothesis

$$H0: \mu_1 = \mu_2$$

expressing the equality of the two population means.

To perform a test of significance for H0, we proceed with the following steps:

1. Decide whether a *one-sided test*, say

$$Ha : \mu_2 > \mu_1$$

or a *two-sided test*,

$$Ha: \mu_1 \neq \mu_2 \qquad \text{is appropriate.}$$

2. Choose a significance level α , a common choice being 0.05.

3. Calculate the *t* statistic,

$$t = \frac{\overline{X}1 - \overline{X}2}{SE\ (\overline{X}1 - \overline{X}2)}$$

where

$$SE_{(\overline{X}1 - \overline{X}2)} = Sp\sqrt{\frac{1}{n1} + \frac{1}{n2}}$$

$$Sp^2 = \frac{(n1-1)S1^2 + (n2-1)S2^2}{n1+n2-2}$$

This test is referred to as a *two-sample t test* and its rejection region is determined using the *t* distribution at ($n_1 + n_2 - 2$) degrees of freedom:

• For a one-tailed test, use the column corresponding to an upper tail area of 0.05 and **H₀** is rejected if

$$t \leq \text{ - tabulated value for Ha:} \mu_1 < \mu_2$$

or $t \geq$ **tabulated value for Ha:$\mu_1 > \mu_2$**

• For a two-tailed test or Ha: $\mu_1 \neq \mu_2$. Use the column corresponding to an upper tail area of 0.025 and **H_0** is rejected if

$t \leq$ - tabulated value   or $t \geq$ tabulated value

Example:. In an attempt to assess the physical condition of joggers, a sample of $n_1 = 25$ joggers was selected and their maximum volume of oxygen uptake (VO2) was measured with the following results:

$\overline{X}_1$= 47.5 mL/kg S_1 = 4.8 mL/kg

Results for a sample of n_2= 26 no joggers were

$\overline{X}_2$= 37.5 mL/kg S_2= 5.1 mL/kg

To proceed with the two-tailed, two-sample t test, we have :.

$Sp^2 = \dfrac{(n1-1)S1^2 + (n2-1)S2^2}{n1+n2-2}$

$Sp^2 = \dfrac{(25-1)(4.8)^2 + (26-1)(5.1)^2}{25+26-2} \quad = 24.56$

$Sp = \sqrt{Sp^2}$

$\quad = \sqrt{24.56} \quad = 4.96$

$SE_{(\overline{X}1 - \overline{X}2)} = Sp\sqrt{\dfrac{1}{n1} + \dfrac{1}{n2}}$

$\qquad = 4.96\sqrt{\dfrac{1}{25} + \dfrac{1}{26}} \quad = 1.39$

It follows that

$t = \dfrac{47.5-37.5}{1.39} \quad = 7.19$

Indicating a significant difference between joggers and no joggers (at 49 df. and α = 0.01, the tabulated *t* value, with an upper tail area of 0.025, is about 2.0).

Example:. Vision, or more especially visual acuity, depends on a number of factors. A study was undertaken in Iraqi to determine the effect of one of these factors: racial variation. Visual acuity of recognition

as assessed in clinical practice has a defined normal value of 20/20 (or zero in log scale). The following summarized data on monocular visual acuity (expressed in log scale) were obtained from two groups:

1. Iraqi males of Arabian origin

$$n_1 = 89$$
$$\overline{X}_1 = -0.20$$
$$S_1 = 0.18$$

2. Iraqi males of Non-Arabian origin

$$n_2 = 107$$
$$\overline{X}_2 = -0.26$$
$$S_2 = 0.13$$

To proceed with a *two-sample t test*, we have

$$Sp^2 = \frac{(n1-1)S1^2 + (n2-1)S2^2}{n1+n2-2}$$

$$Sp^2 = \frac{(89-1)(0.18)^2 + (107-1)(0.13)^2}{89+107-2} = 0.0239$$

$$Sp = \sqrt{Sp^2}$$

$$= \sqrt{0.0239} = 0.155$$

$$SE_{(\overline{X}1 - \overline{X}2)} = Sp\sqrt{\frac{1}{n1} + \frac{1}{n2}}$$

$$= 0.155\sqrt{\frac{1}{89} + \frac{1}{107}} = 0.022$$

$$t = \frac{(-0.20)-(-0.26)}{0.022} = 2.73$$

The result indicates that the difference is statistically significant beyond the 0.01 level (at α = 0.01 , and for a two-sided test "α = 0.005" the cut point is 2.58 for high df. values).

Example:. The extent to which an infant's health is affected by parental smoking is an important public health concern. The following data are the urinary concentrations of cotinine (a metabolite of nicotine);

measurements were taken both from a sample of infants who had been exposed to household smoke and from a sample of unexposed infants.

Unexposed ($n_1 = 7$)// 8 , 11 , 12 , 14 , 20 , 43 , 111

Exposed ($n_2 = 8$)// 35 , 56 , 83 , 92 , 128 , 150 , 176 , 208

The statistics needed for our ***two-sample t test*** are:

1. For unexposed infants:

$$n_1 = 7$$
$$\overline{X}_1 = 31.29$$
$$S_1 = 37.07$$

2. For exposed infants:

$$n_2 = 8$$
$$\overline{X}_2 = 116.00$$
$$S_2 = 59.99$$

To proceed with a ***two-sample t test***, we have

$$Sp^2 = \frac{(n1-1)S1^2 + (n2-1)S2^2}{n1+n2-2}$$

$$Sp^2 = \frac{(7-1)(37.07)^2 + (8-1)(59.99)^2}{7+8-2}$$

$$= \frac{33436.71}{13} = 2572.0546$$

$$Sp = \sqrt{Sp^2}$$

$$= \sqrt{2572.0546} = 50.72$$

$$SE_{(\overline{X}1 - \overline{X}2)} = Sp\sqrt{\frac{1}{n1} + \frac{1}{n2}}$$

$$= 50.72\sqrt{\frac{1}{7} + \frac{1}{8}} = 26.25$$

$$t = \frac{(31.29)-(116)}{26.25} = -3.23$$

The result indicates that the difference is statistically significant beyond the 0.01 level (at $\alpha = 0.01$, and for a ***two-sided test*** " $\alpha = 0.005$ " the cut point is - 3.012 for 13 df.).

- $t_{cal.} < - t_{tab.}$

- $3.23 < - 3.012$ The difference is statistically significant

Question:.

A study was conducted to investigate whether oat bran cereal helps to lower serum cholesterol in men with high cholesterol levels. Fourteen men were randomly placed on a diet which included either oat bran or cornflakes ; after two weeks their low – density – lipoprotein cholesterol levels were recorded. Each man was then switched to the alternative diet. After a second two – week period, the LDL cholesterol level of each person was recorded again. Test to compare the means LDL cholesterol level. Name the test and state clearly your null and alternative hypotheses and choice the test size.

| LDL (mmol/L) ||
Cornflakes	Oat Bran
4.61	3.84
6.42	5.57
5.40	5.85
4.54	4.80
3.98	3.68
3.82	2.96
5.01	4.41
4.34	3.72
3.80	3.49
4.56	3.84
5.35	5.26
3.89	3.73
2.25	1.84
4.24	4.14