Two - way analysis of variance

This is the simplest level of multivariate ANOVA. Here we have a second treatment variable of interest. What if we wanted to know if different sexes reacted to increasing levels of a drug ?

We now need three different indexes :.

i for different treatment groups (or **A** factor).

j for different block (or **B** factor).

k for different observations within blocks.

The number of A factor levels = **a**

The number of B factor levels = **b**

	A				Sum	Mean
B	**1**	**2 ………**	**i ………**	**a**		
1	X_{11}	X_{21} ………	X_{i1} ………	X_{a1}	$\mathbf{X_{\cdot 1}}$	$\overline{\mathbf{X}}_{\cdot 1}$
2	X_{12}	X_{22} ………	X_{i2} ………	X_{a2}	$\mathbf{X_{\cdot 2}}$	$\overline{\mathbf{X}}_{\cdot 2}$
.	.	.	.	.	**.**	**.**
j	X_{1j}	X_{2j} ………	X_{ij} ………	X_{aj}	$\mathbf{X_{\cdot j}}$	$\overline{\mathbf{X}}_{\cdot j}$
.	.	.	.	.	**.**	**.**
b	X_{1b}	X_{2b} ………	X_{ib} ………	X_{ab}	$\mathbf{X_{\cdot b}}$	$\overline{\mathbf{X}}_{\cdot b}$
Sum	$\mathbf{X_{1.}}$	$\mathbf{X_{2.}}$	$\mathbf{X_{i.}}$	$\mathbf{X_{a.}}$	$\mathbf{X_{..}}$	
Mean	$\overline{\mathbf{X}}_{1.}$	$\overline{\mathbf{X}}_{2.}$	$\overline{\mathbf{X}}_{i.}$	$\overline{\mathbf{X}}_{a.}$		$\overline{\mathbf{X}}_{..}$

ANOVA table

S.O.V.	df.	SS	MS	$F_{statistic}$	$F_{tab.}$ 0.01	$F_{tab.}$ 0.05
A(between A levels)	a-1	SSA	MSA	F_A=MSA/MSE		
B(between B levels)	b-1	SSB	MSB	F_B=MSB/MSE		
Error(within)	(a-1)(b-1)	SSE	MSE			
Total	ab-1	SST				

SST= $\sum\sum(Xij - X..)^2$

$= \sum\sum Xij^2 - CF$

CF = $\frac{X..^2}{ab}$

SSA = $\sum\sum(Xi. - X..)^2$

$= \frac{\sum Xi.^2}{b} - \text{CF.}$

SSB = $\frac{\sum X.j^2}{a} - \text{CF.}$

SSE = $\sum\sum(Xij - Xi. - X.j - X..)^2$

= **SST – SSA – SSB**

SST = SSA + SSB + SSE

Notes:.

1- The stander error of **a** mean is :.

$$S_{Xi.} = \sqrt[2]{\frac{MSE}{b}}$$

2- The stander error of **b** mean is :.

$$S_{X.j} = \sqrt[2]{\frac{MSE}{a}}$$

3- The stander error of the different between two means of **A** is :.

$$S_{X1.-X2.} = \sqrt[2]{\frac{2MSE}{b}}$$

4- The stander error of the different between two means of **B** is :.

$$S_{X.1-X.2} = \sqrt[2]{\frac{2MSE}{a}}$$

5- H_{0A} : $X_{1.} = X_{2.} = \ldots\ldots = X_{i.}$ *vs* H_{aA}: at least one mean of A is not equal to the others.

H_{0B} : $X_{.1} = X_{.2} = \ldots\ldots\ldots = X_{.j}$ *vs* H_{aB}: at least one mean of B is not equal to the others.

Example:.

A study was conducted to investigate of effect cars movement in Karbala city on air pollution. Five samples are taken from different sites at four different months. The table below provides data on quantitative of sold material (mg/m^3). Test if there is difference in pollution level between the sites or between the months.

Air pollution in five different sites from Karbala city.

Time A	**Location : B**					***Sum***
	1	**2**	**3**	**4**	**5**	
Oct.	76	67	81	56	51	***331***
Jan.	82	69	96	59	70	***376***
May.	68	59	67	54	42	***290***
Sept.	63	56	64	58	37	***278***
Sum	***289***	***251***	***308***	***227***	***200***	***1275***

Answer:.

$X_{..} = 1275$

$X_{1.} = 331$ $X_{2.} = 376$ $X_{3.} = 290$ $X_{4.} = 278$

$X_{.1} = 289$ $X_{.2} = 251$ $X_{.3} = 308$ $X_{.4} = 227$ $X_{.5} = 200$

$$\text{CF.} = \frac{(X_{..})^2}{ab} = \frac{(1275)^2}{4\times 5} = \mathbf{81281.25}$$

$$\textbf{SST} = \sum\sum Xij^2 - CF$$

$$= [(76)^2 + (67)^2 + \ldots + (37)^2] - 81281.25 = \boldsymbol{3751.75}$$

$$\textbf{SSA} = \frac{\sum Xi.^2}{b} - \text{CF.}$$

$$= \frac{331^2 + 376^2 + 290^2 + 278^2}{5} - 81281.25 = \boldsymbol{1182.95}$$

$$\textbf{SSB} = \frac{\sum X.j^2}{a} - \text{CF.}$$

$$= \frac{289^2 + 251^2 + 308^2 + 227^2 + 200^2}{4} - 81281.25 = \boldsymbol{1947.50}$$

SSE = SST – SSA – SSB

= 3571.75 – 1182.95 – 1947.50 = ***441.30***

ANOVA table

S.O.V.	df.	SS	MS	$F_{stat.}$	$F_{tab.0.01}$	$F_{tab.0.05}$
A	3	1182.95	394.32	F_A=10.72	5.95	3.49
B	4	1947.50	486.875	F_B=13.24	5.41	3.26
Error	12	441.30	36.775			
Total	**19**	**3571.75**				

The results indicate there is significant difference in pollution degree between the months when we compare F_A with $F_{(3, 12)}$ at α : 0.01 and 0.05 levels. In addition to this, there is significant difference in pollution degree between the sites when we compare F_B with $F_{(4, 12)}$ at α : 0.01 and 0.05 levels.

Now, can you determine the site is more than the others or the month is more than the others in pollution degree???

The answer of the above question will study in the following lecture.