The Multiple Comparisons

There are many methods for multiple-comparisons. Most are for pairwise comparisons of group means, to determine which are significantly different from which others. Other methods, though, are for more specialized purposes (*e.g.* ***comparing each of several treatments to a control***) or allow testing of more - general hypotheses contrasting sets of group means (***as is often done in preplanned "contrasts").***

The various methods differ in how well they properly control the overall significance level and in their relative power; some, such as the popular "***Duncan's multiple range test***" **do not** control the overall significance level. The ones described in this handout all adequately control overall significance and are either easy to use or powerful. They are:.

- **Bonferroni** : extremely general and simple, but often not powerful.
- **Tukey's** :the best for all-possible pairwise comparisons when sample sizes are unequal or confidence intervals are needed; very good even with equal samples sizes without confidence intervals.
- **Stepdown**: the most powerful for all possible pairwise comparisons when sample sizes are equal.
- **Dunnett's** :for comparing one sample ("***control***") to each of the others, but not comparing the others to each other.

• **MCB** : compares each mean to the "best" (largest or smallest, as you specify) of the other means.

• **Scheffé's** :for unplanned contrasts among sets of means.

• **Fisher's (LSD):**Is a method for comparing treatment group means after the ANOVA null hypothesis of equal means has been rejected using the ANOVA F-test. If the F-test fails to reject the null hypothesis this procedure should not be used.

Least Significant Differences (LSD)

To find the LSD value :.

- ❖ Calculate **MSE** or read from ANOVA table. ***Note:MSE = S_p^2***
- ❖ Calculate **SED** *" standard error deviation"*.

SED = $\sqrt{\frac{2MSE}{r}}$ or $\sqrt{MSE \times \frac{1}{n1} + \frac{1}{n2}}$

- ▪ Use r ,the number of replications, if the required SED is between all the means of a given experiment. In addition to the observation number of all sample is equal.
- ▪ Use $[\frac{1}{n1} + \frac{1}{n2}]$ for SED is between two specified means, and $n_1 \neq n_2$. In other word, the observation number of samples is different.

- ❖ Find the table t , using **df_{error}** with required level of significance (two – tailed).
- ❖ Calculate **LSD = t_{table} × SED.**

❖ Compare this **LSD** with the differences between the pairs of means and make a decision as to which pairs are significantly different.

Example :.

Let s assume we have 4 treatment groups A, B, C and D. the summary statistics for groups are :.

Group	$\bar{y}$	S	n
A	74.3	5.6	10
B	82.8	5.1	10
C	77.8	5.3	10
D	82.9	4.6	10

The ANOVA table for this data is :.

S.O.V.	df	SS	MS	$F_{statistics}$	$F_{table\ 0.05}$
Between samples	3	523.7	174.57	6.55	2.888
Within samples	36	959.58	26.65		
Total	39	1483.28			

Since the F_{table} less than $F_{statistics}$ at o.o5 level, we reject the null hypothesis $H_0 : \mu_A = \mu_B = \mu_C = \mu_D$.

At this point we are interested in doing pairwise comparisons of the means. That is , we want to test hypotheses of the sort $H_0 : \mu_A = \mu_B$, $H_0 : \mu_A = \mu_C$, etc. The LSD method for testing the hypothesis H_0 proceeds as follows :.

$$LSD = t_{(0.05/2)\,,\,\text{df error}}\sqrt{\frac{2MSE}{r}}$$

$$t_{(0.025),\,39} = 2.0231$$

$$SED = \sqrt{\frac{2\times 26.65}{10}}$$

$$LSD = 2.0231\sqrt{\frac{2\times 26.65}{10}}$$

$$= 4.67$$

If $|A-B| \geq$ LSD then we reject the null hypothesis. For our example $|A-B| = 8.5$ which is greater than 4.67 so we reject H_0 at the 0.05 level.

We then continue to test all pairs of interest. In this case the LSD is the same for all pairs because $n_A = n_B = n_C = n_D$. Thus $LSD_{A,B} = LSD_{A,C} = \ldots = LSD_{C,D} = 4.67$ and we compare all pairwise differences in the means to 4.67 . Here are the absolute pairwise differences and the results of Fisher s LSD :.

$|A-B| = 8.5 \geq 4.67$ reject $H_0 : \mu_A = \mu_B$.

$|A-C| = 3.5 < 4.67$ do not reject $H_0 : \mu_A = \mu_C$.

$|A-D| = 8.6 \geq 4.67$ reject $H_0 : \mu_A = \mu_D$.

$|B-C| = 5.0 \geq 4.67$ reject $H_0 : \mu_B = \mu_C$.

$|B-D| = 0.1 < 4.67$ do not reject $H_0 : \mu_B = \mu_C$.

$|C-D| = 5.1 \geq 4.67$ reject $H_0 : \mu_C = \mu_D$.

Example :.

A trial with 5 treatments was replicated 4 times. Given the following data , find the pairs of means with significant differences. Tested at 0.05 level.

Treatment :.	A	B	C	D	E
Mean :.	8.0	6.5	3.0	4.0	5.4

MSE = 4.53 and $df_{error} = 15$

Calculations :.

* **SED** = $\sqrt{\frac{2MSE}{r}} = \sqrt{\frac{2\times4.53}{4}}$

* $\mathbf{t}_{(0.05/2),\,15} = 2.131$ at 0.05 level.

* $\mathbf{t}_{(0.01/2),\,15} = 2.947$ at 0.01 level.

* $\mathbf{LSD}_{\text{at } 0.05} = t_{(0.05/2),\,15}\sqrt{\frac{2MSE}{r}}$

$= 2.131\sqrt{\frac{2\times4.53}{4}}$

$= 3.21$

<u>The difference between means</u>

A – B = 1.5 $<$ 3.21 No significantly different.

A – C = 5 $\geq$ 3.21 Significantly different.

A – D = 4 $\geq$ 3.21 Significantly different.

A – E = 2.6 $<$ 3.21 No significantly different.

B – C = 3.5 $\geq$ 3.21 Significantly different.

B – D = 2.5 $<$ 3.21 No significantly different.

B – E = 1.1 $<$ 3.21 No significantly different.

C – D = 1.0 $<$ 3.21 No significantly different.

C – E = 2.4 $<$ 3.21 No significantly different.

D – E = 1.4 $<$ 3.21 No significantly different.

That is , at the 0.05 level , mean of A is significantly different to **C** and **D** ; mean of **B** is significantly different to **C**; no other means are significantly different.

$$LSD_{0.01} = t_{(0.01/2),\,15}\sqrt{\frac{2MSE}{r}}$$

$$= 2.947\sqrt{\frac{2\times 4.53}{4}} = 4.435$$

<u>The different between means</u>

A – B = 1.5 $<$ 4.435 No significantly different.

A – C = 5 $\geq$ 4.435 Significantly different.

A – D = 4 $<$ 4.435 No significantly different.

A – E = 2.6 $<$ 4.435 No significantly different.

B – C = 3.5 $<$ 4.435 No significantly different.

B – D = 2.5 $<$ 4.435 No significantly different.

B – E = 1.1 $<$ 4.435 No significantly different.

C – D = 1.0 $<$ 4.435 No significantly different.

C – E = 2.4 $<$ 4.435 No significantly different.

D – E = 1.4 $<$ 4.435 No significantly different.

At the 0.01 level , mean of A is significantly different to **C** ; no other means are significantly different.

***Example* :.** (ANOVA two – way)

In the example of air pollution in five different sites from Karbala city. The ANOVA table for this example is :.

S.O.V.	df	SS	MS	$F_{statistics}$	$F_{table\ 0.01}$
$A_{(time)}$	3	1182.95	394.32	F_A=10.72	3.49
$B_{(location)}$	4	1947.50	486.875	F_B=13.24	3.26
Error	12	441.30	36.775		
Total	19	3571.75			

The summary statistics for groups are :.

Group A (Months)	$\overline{X}$	n
Oct.	66.2	5
Jan.	75.2	5
May.	58	5
Sept.	55.6	5

Group B (Location)	$\overline{Y}$	n
Location 1	72.25	4
Location 2	62.75	4
Location 3	77	4
Location 4	56.75	4
Location 5	50	4

Tested that at α : 0.01

Answer :.

From above data

MSE = 36.775

$n_A = 5$

$n_B = 4$

$df_{error} = 12$

$$SED_A = \sqrt{\frac{2MSE}{n}}$$

$$= \sqrt{\frac{2 \times 36.775}{5}}$$

$t_{(0.01/2),\,12} = 3.055$

$$LSD_A = t_{(0.01/2),\,12}\sqrt{\frac{2MSE}{n}}$$

$$= 3.055\sqrt{\frac{2 \times 36.775}{5}}$$

$$= 11.717$$

The difference between A means (the months).

$|Oct. - Jan.| = 9 < 11.717$ No significantly difference

$|Oct. - May.| = 8.2 < 11.717$ No significantly difference

$|Oct. - Sept.| = 10.6 < 11.717$ No significantly difference

$|Jan. - May.| = 17.2 \geq 11.717$ Significantly difference

$|Jan. - Sept.| = 19.6 \geq 11.717$ Significantly difference

$|May. - Sept.| = 2.4 < 11.717$ No significantly difference

That is , at the 0.01 level , means of months are significantly difference only between Jan. to May. , and Jan. to Sept. No other means are significantly difference.

$$SED_B = \sqrt{\frac{2MSE}{n}}$$

$$= \sqrt{\frac{2 \times 36.775}{4}}$$

$$LSD_B = t_{(0.01/2),\,12}\sqrt{\frac{2MSE}{n}}$$

$$= 3.055\sqrt{\frac{2\times 36.775}{4}}$$

$$= 13.1$$

The difference between B means (the locations).

$|1-2| = 9.5 \quad < \quad 13.1$ No significantly difference

$|1-3| = 4.5 \quad < \quad 13.1$ No significantly difference

$|1-4| = 15.5 \quad \geq 13.1$ Significantly difference

$|1-5| = 22.25 \quad \geq 13.1$ Significantly difference

$|2-3| = 14.25 \quad \geq 13.1$ Significantly difference

$|2-4| = 6 \quad < \quad 13.1$ No significantly difference

$|2-5| = 12.75 \quad < \quad 13.1$ No significantly difference

$|3-4| = 20.25 \quad \geq 13.1$ No significantly difference

$|3-5| = 27 \quad \geq 13.1$ No significantly difference

$|4-5| = 6.75 \quad < \quad 13.1$ No significantly difference